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Finite Wordlength in Digital Filters: A Review

by
H.J. Butterweck
J.H.F. Ritzerfeld
M.J. Werter

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ABSTRACT

A review is presented of recent work on quantization and overflow effects in digital filters. These unwanted non-linear phenomena include parasitic oscillations (limit cycles) and quantization noise. Modern stabilization methods and noise optimization strategies are discussed. A comprehensive bibliography contains the relevant original contributions dealing with the analysis of various finite wordlength effects and measures to reduce or avoid them.

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Finite wordlength effects in digital filters - a review

I. Introduction

In most applications signal processing in digital filters is intended to be performed in the form of *linear* operations, which for the important class of time-invariant systems are of the convolution type. The digital encoding of the various signals, however, implies that in general the required linearity can be achieved only to a certain degree. Fortunately, the deviation from the linear behaviour can be made arbitrarily small through choosing sufficiently long binary words. Yet there remain typical *finite-wordlength effects* that cause an actual digital filter to behave as a (weakly) *nonlinear* system.

Contrary to the finite wordlength of the signals to be processed the finite wordlength of the *filter coefficients* does not affect the linearity of the filter behaviour. This effect only amounts to restrictions on the linear filter characteristics, resulting in discrete grids of pole-zero patterns. Once a filter design with some combination of permitted coefficients¹ meets the required specifications (with regard to amplitude and phase characteristics) the actual filter performance differs from that predicted by linear theory only as to the previously mentioned nonlinear finite-wordlength effects. These effects, which divide into those due to "*signal quantization*" and those due to "*overflow*", form the subject matter of the present paper. Our interest in coefficient quantization is only indirect and stems from some relation between the sensitivity of the filter characteristics to parameter variations on the one hand and the generation of quantization noise due to signal quantization on the other. This relation states that in general low-sensitivity structures (allowing short coefficient words) are distinguished by low noise levels [11]-[17].

The majority of quantization and overflow phenomena can be derived from a simple model, in which appropriate nonlinear, memoryless components (NL) are inserted into an otherwise linear, idealized digital system.

¹ In the binary format a coefficient can only assume a value $p/2^n$ with $p \in \mathbb{Z}$ and $n \in \mathbb{N}$.

A typical NL characteristic is shown in Fig. 1.1; it is characterized by a fixed-point number representation (with 3 bits yielding 2^3 different signal levels), rounding R as quantization, and saturation as overflow correction.

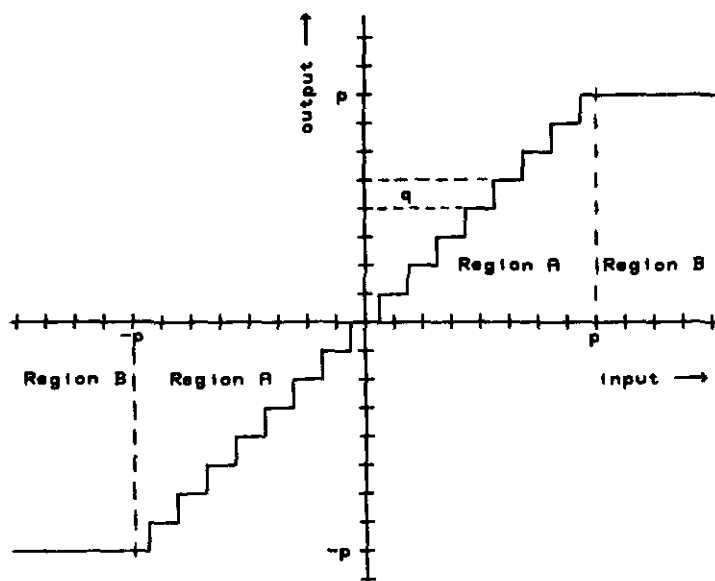


Fig. 1.1. Characteristic of a finite-wordlength nonlinearity.

Also other combinations can be conceived and will be studied in due course. Common to all these nonlinear characteristics are the following properties: a) For inputs whose magnitudes are smaller than p (Region A) the output is close to the input; the difference is that the former is machine-representable, while the latter is unrestricted; b) for all inputs whose magnitude is greater than p (Region B) the magnitude of the output cannot exceed p . Region A models quantization after multiplication by a constant factor, whereas Region B represents the correction required in connection with adder overflow.

The question where the various nonlinearities NL have to be inserted into the linear network, can be straightforwardly answered for any structure and its pertinent computation scheme. Care must be taken that every *feedback loop* must contain at least one NL element to avoid ever-increasing word-lengths. FIR filters without loops do not strictly require such elements; quantization and overflow correction is, however, often applied for intentional wordlength limitation. In any case, the AD-converter preceding a digi-

tal filter ultimately causes every digitally realized system with analogue input/output terminals to exhibit more or less nonlinear signal distortion.

In a common approximation, *quantization* and *overflow* are not only conceptually decoupled, but also analytically treated as *independent effects*. This implies that for large signals the fine quantization structure is neglected and the Region A-part of the nonlinear characteristic is replaced by a 45° straight line. Apparently this approximation can only be justified if the total number of quantization steps is large enough or, in other words, the binary words are sufficiently long. Even for this extreme case several authors have queried the validity of the decoupling approximation [18]-[22]. Indeed, there are overflow effects that can only be properly understood in connection with the quantization fine structure. As an example, consider a filter initially in the zero state and then excited by a short, strong pulse such that overflow occurs at some point inside the filter. Assume that the idealized (quantization-free) filter asymptotically returns to equilibrium (zero state), which implies "overflow stability" (cf. Section III).

Apparently, the filter has "forgotten" the overflow after a sufficiently long time. With quantization, the situation is not as simple: before excitation, the filter might (necessarily) oscillate in a limit cycle mode, while after overflow the filter does not recover to the zero state but again enters a limit cycle. The mode of oscillation can, however, be completely different from the former one. Because the filter never forgets the overflow, it has apparently to be considered as unstable.

Recently, chaotic overflow oscillations have been observed [23]. Also in that case the quantization has been neglected in the first instance. Taking the fine structure of the NL characteristic into account, the filters under consideration become finite-state machines with strictly periodic (non-chaotic) oscillations.

These examples belong to a small group of exceptional phenomena where the decoupling assumption fails even for a large dynamic range (long binary words). For most effects to be treated in this paper it is valid with sufficient accuracy.

The simple NL-model with a characteristic like that of Fig. 1.1 does not apply to all finite-wordlength mechanisms. This is particularly true for all types of "*controlled rounding*" (CR), in which the treatment of the least significant bit is not controlled by the signal to be quantized but by another signal. So it is often devised that an external, mostly stochastic signal controls the quantization or that an internal signal within the filter performs that task. More complicated schemes leave the decision about the rounding direction (upwards or downwards) to more than one control signal, one of which may be the signal to be quantized. All these methods are in current use to suppress quantization limit cycles and will be discussed in Section IV. We note that also a controlled overflow correction is conceivable, although attempts in this direction have not yet been reported.

II. Quantization and overflow characteristics

Returning to the NL model we have still to review other *characteristics for quantization and overflow* than that shown in Fig. 1.1. Although less frequently used than its fixed-point counterpart, a *floating-point realization* of a digital filter often deserves consideration. Also for this arithmetic finite-wordlength effects have to be reckoned with, including limit cycles [24] and quantization noise [25]. A completely different design approach of a more recent date makes use of "*residue arithmetic*", (a number-theoretical tool). The associated finite-wordlength effects have not yet drawn too much attention [26]-[29].

For conventional fixed-point arithmetic we can mainly choose from three *quantization schemes* with specific individual merits: (a) rounding R, (b) magnitude truncation MT, (c) value truncation VT. Each method is characterized by a peculiar instruction rule concerning the direction of quantization (upwards or downwards): (a) for R towards the nearest machine-representable number (b) for MT towards zero (c) for VT always downwards. Let x and $Q(x)$ denote the unquantized and quantized number, respectively, and let further $\epsilon(x) = Q(x) - x$ denote the "quantization error", and q the quantization step size, then we have

$$\begin{aligned} |\epsilon_R(x)| &\leq q/2 \\ |\epsilon_{MT}(x)| &< q \\ |\epsilon_{VT}(x)| &< q \end{aligned} \tag{2.1}$$

which admits the conclusion that rounding is the most attractive form of quantization with regard to the average error signal amplitude. The specific advantage of magnitude truncation lies in its inherent capability of limit cycle suppression (cf. Section IV), that follows from energy considerations in connection with the basic MT property $|Q_{MT}(x)| \leq |x|$. Finally, value truncation is the natural quantization method for a two's complement arithmetic. Its formal treatment is similar to that of rounding due to the simple relation

$$Q_{VT}(x) = Q_R(x - q/2) \tag{2.2}$$

stating that VT yields the same results as R after adding the constant signal $q/2$ to the unquantized signal.

Comparing the two main quantization schemes "rounding" and "magnitude truncation" we observe fundamental differences in their nonlinear signal processing behaviour, which follow from their error characteristics $\epsilon(x)$, cf. Fig. 2.1.

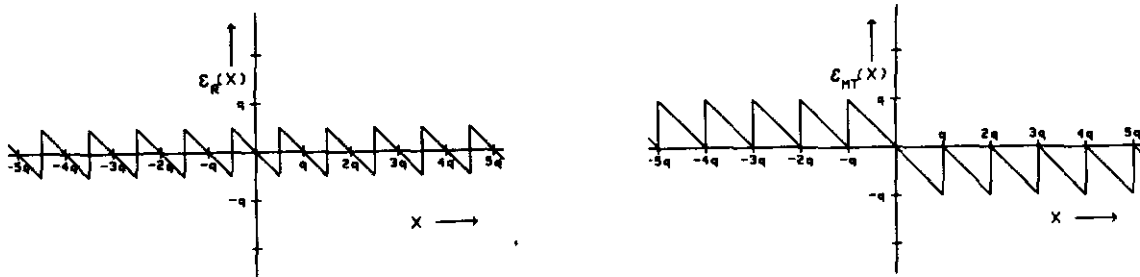


Fig. 2.1. Quantization error $\epsilon(x)$ for rounding R and magnitude truncation MT.

It is true that both characteristics are strictly deterministic, i.e. with every x a unique error signal $\epsilon(x)$ is associated. Nevertheless, we are inclined to attribute "quasi-random" features to the rounding characteristics in the following sense. If $x(k)$ is assumed² to represent a stationary random process characterized by a probability density function $P(x)$ and an autocorrelation function $s_{xx}(m) = E\{x(k) x(k-m)\}$ this process is transformed by the rounding error characteristic into another process $\epsilon(k)$, which "almost always" has white-noise character with $s_{\epsilon\epsilon}(m) = q^2/12 \delta(m)$ as well as a uniform probability distribution in the interval $-\frac{q}{2} \leq \epsilon \leq \frac{q}{2}$. This property is the basis for the well-established *white-noise model* of the *rounding error* [30], which we also adopt in this paper. The reliability of this model improves with increasing level of the signal $x(k)$ and with increasing spread of its power spectrum. It *fails* completely if $x(k)$ varies *periodically*, associated with a line power spectrum. Then also $\epsilon(k)$ is periodic and, hence, not noisy. Such a periodicity applies e.g. when a recursive filter oscillates in a *limit cycle* mode (cf. Section IV).

²The symbol k denotes the discrete time variable.

To analyze the corresponding error characteristic for *magnitude truncation* we first split it into two parts according to Fig. 2.2. The first part resembles the $\epsilon_R(x)$ characteristic and will henceforth be referred to as the "quasi-rounding" component $\epsilon_{QR}(x)$ of magnitude truncation. The white-noise model of the rounding error likewise applies to quasi-rounding, so that R and MT essentially differ in the second part of the MT error characteristic, the so-called "sign-part" $\epsilon_{SGN}(x)$, (cf. Fig. 2.2).

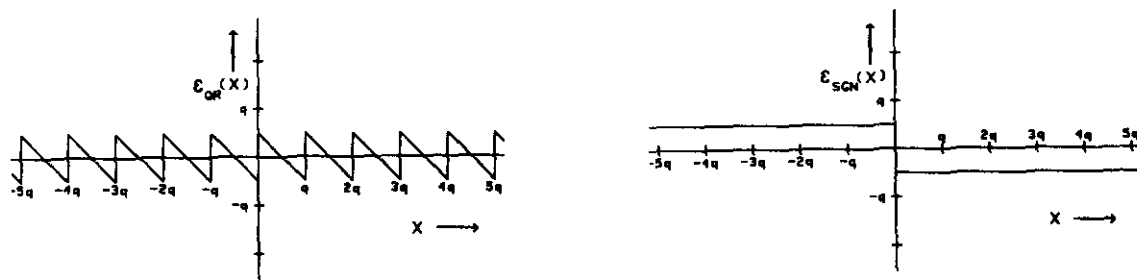


Fig. 2.2. Decomposition of the quantization error for magnitude truncation into a quasi-rounding and a sign part.

As to their signal-processing behaviour, the quasi-rounding and the sign part are basically dissimilar. While the former part lends itself to a modelling as an *additive* (white-)noise source³, the latter remains an essentially non-linear component whose output is strongly correlated with the input signal. In some applications a straight line through the origin with an appropriate negative slope can be advantageously split off from $\epsilon_{SGN}(x)$, resulting in slight modifications of the filter coefficients and, ultimately, in effects of *detuning* (including Q-factor modifications). Apparently such detuning is level-dependent and decreases with increasing signal amplitude. What remains is a pure *nonlinear signal degradation*, that leads to a number of interference phenomena (including crosstalk [31]-[34]) and that has to be interpreted ordinary distortion in the audio region.

³If the system contains more than one quantizer, the model is extended such that the various noise sources are uncorrelated.

While quantization has to be accepted as an unavoidable concomitant of any digital signal processing, the situation is less constraining with respect to overflow. Obviously, overflow can be completely avoided through using sufficiently small input signals: For a given impulse response (considered between the input terminal and a node of potential overflow) and for a prescribed overflow level an upper bound for the input signal can easily be derived [35].

Nevertheless, it is common practice to accept a small risk of overflow, occurring for very unfavourably chosen excitations. Thus the dynamic range of a filter is better exploited, ultimately resulting in a lower quantization noise level. This mild "scaling policy" consciously tolerates a small nonzero probability of overflow. So, infrequent overflows and accompanying interruptions of normal operation are accepted under the obvious tacit assumption that after each overflow the normal operation recovers, preferably with high speed.

The required recovery automatically leads to the paramount problem of overflow stability. To discuss this item we assume that the underlying idealized, linear system is stable and that quantization can be neglected (decoupling assumption). Then the stability problem is attacked in two steps, (a) under zero-input conditions, (b) under nonzero-input conditions. Stability according to (a) is defined as absence of spontaneous oscillations, particularly of periodic nature. A system stable in this sense is asymptotically (from a certain time instant k_0) overflow-free. Then it behaves linearly and (exponentially) approaches the equilibrium point in which all state variables become zero. Stability according to (b), the so-called "forced-response stability" is defined for a certain class U_0 of input signals $u(k)$. Such signals are defined with the aid of the idealized linear system and characterized by the property that for at least one initial condition the overflow threshold is never reached. The filter with overflow correction is then called "forced-response stable" if for any $u(k) \in U_0$ and any initial condition the response asymptotically ($k \rightarrow \infty$) approaches the waveform of the linear counterpart.

So for the given class of input signals the actual filter eventually "forgets" former overflows and becomes overflow-free. Clearly, forced-response stability is a stronger condition than zero-input stability and includes the latter.

If the system is excited with a rather irregular waveform, zero-input stability will often suffice; only for periodic waveforms the stronger condition is strictly required.

Mainly three overflow characteristics have been proposed: (a) *saturation* (b) *zeroing* (c) *two's complement*, cf. Fig. 3.4 with $v=0$. Saturation yields the smallest deviation from the normal operation, although during overflow the filter becomes more or less inoperative. It has also the best stability properties (cf. Section III). Zeroing means that the output is set to zero, if the input exceeds the overflow threshold; it can be easily generalized to reset *all* states, when *one* state exhibits overflow. Two's complement overflow amounts to a periodic continuation of the 45° straight line; its advantage lies in the automatic correction of intermediate overflows. With regard to stability it is the least favourable overflow correction so that the choice of the linear circuit is more restricted than for the other characteristics.

We conclude this section with a few remarks on the aim and organization of the paper. First of all, a comprehensive *bibliography* covers all nonlinear finite-wordlength effects in one-dimensional digital filters published in recognized journals and conference proceedings. Multidimensional filters and coefficient quantization have been left out of consideration. The text has been written in awareness of existing review articles [3]-[10] and should especially be viewed as an extension of Claassen's (et al) paper of 1976; in fact, it is a progress report covering the past twelve years. It should further be noted that not all aspects are treated with the same elaborateness. So, only a brief discussion is devoted to structure optimization with respect to quantization noise, mainly due to an exhaustive treatment of this subject in two recent textbooks [1],[2].

The references from [424] onward are recent contributions (published in the years 1987 and 1988) to non-linear effects in digital filters, which were added after the manuscript of this report was completed, and as such are not referenced in the text.

For ease of reference, a bibliography in alphabetical order of all authors is added.

III. Overflow oscillations

In *recursive* filters, quantization and overflow can lead to instabilities, even if the underlying linear filter is designed to behave stable. Instabilities due to quantization ("limit cycles") lead to relatively small deviations from the linear behaviour. While these effects will be treated in the next section, we now deal with those instabilities that are related to register *overflow*. The associated *oscillations* have large amplitudes; because of their disastrous effects on the filter behaviour they have to be absolutely avoided. One of the main factors determining their occurrence is the "overflow characteristic" (i.e. the way overflow is corrected), of which we treat the three commonly used types (a) saturation (b) zeroing (c) two's complement.

A. Zero-input oscillations

We begin with a study of *overflow oscillations* [38]-[69] in the original sense, i.e. for an otherwise unexcited digital system. In addition to this "zero-input" condition we assume that (a) overflow and quantization can be treated independently ("decoupling assumption") and (b) overflow correction is only required for signals entering a delay element. The latter assumption excludes all structures where intermediate overflows occur. For sake of conciseness, we restrict the following discussion to second-order sections with complex poles. Compared with real poles, complex conjugate pole-pairs generally favour all forms of parasitic oscillations (particularly for high Q-values) and thus deserve special consideration. In due course, we summarize more general results for higher-order sections and without reference to complex pole pairs.

The 2×1 state vector $\underline{x} = (x_1, x_2)^T$ in a second-order system satisfies the fundamental difference equation

$$\underline{x}(k+1) = F(A \underline{x}(k)) \quad (3.1)$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

denotes the *system matrix*, while $F(\cdot)$ stands for the overflow characteristic.

It is understood that $[F(A \underline{x})]_i = F(A \underline{x})_i$, i.e. the individual components of $A \underline{x}$ undergo the same memoryless and local (i.e. not controlled by other signals) overflow correction. The question to be analyzed is: Under which circumstances (choice of A , F and initial conditions) does (or does not) (3.1) admit periodic solutions?

Due to the overflow bound, which is henceforth normalized to unity, the state variables satisfy the condition $|x_i| \leq 1$, resulting in a state vector confined to the interior of the *unit square* (cf. Fig. 3.1). Without overflow (i.e. as long as $|x_i| \leq 1$) the solution of (3.1) is found as

$$\underline{x}(k) = \text{Re}\{\tilde{X}(\underline{\xi}_r + j\underline{\xi}_i)e^{[(r+j\Omega)k+j\tilde{\varphi}]}\} = \tilde{X}e^{rk}[\underline{\xi}_r \cos(\Omega k + \tilde{\varphi}) - \underline{\xi}_i \sin(\Omega k + \tilde{\varphi})], \quad (3.2)$$

where $e^{r \pm j\Omega}$ denotes the complex eigenvalues of A and $\underline{\xi}_r \pm j\underline{\xi}_i$ denotes the pertinent eigenvectors. It is tacitly assumed that $r < 0$, expressing linear stability. Further, without loss of generality, the real and imaginary parts $\underline{\xi}_r$, $\underline{\xi}_i$ of the suitably normalized eigenvector are assumed to be orthogonal, i.e. $\underline{\xi}_r^T \underline{\xi}_i = 0$. (This freedom is provided by the indeterminacy of the complex magnitude of any eigenvector). Finally, the constants of integration $(\tilde{X}, \tilde{\varphi})$ are determined by the initial conditions.

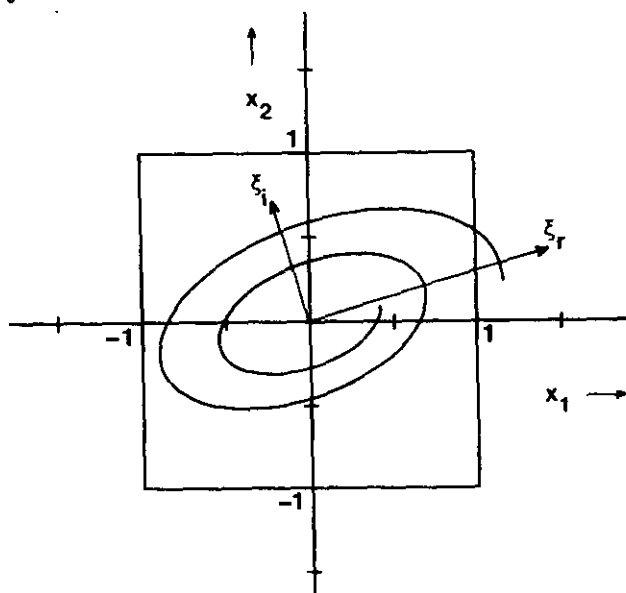


Fig. 3.1. Trajectory of the state vector $\underline{x}(k)$ in the phase plane and the overflow boundary.

If, for the time being, time k is viewed as a continuous variable, $\underline{x}(k)$ describes a trajectory in the phase plane. For the (unrealizable) case $r=0$ this would be an *ellipse* with main axes in the direction of \underline{e}_r and \underline{e}_i . For $r < 0$ (corresponding to poles *inside* the unit circle), we obtain a nonclosed, ellipse-like curve spiralling towards the origin, cf. Fig. 3.1.

Of course, these results only apply to the digital filter as long as overflow does not occur ($|x_i| \leq 1$). In general, this condition is not met for all initial conditions $\underline{x}(0)$ inside the unit square. Only the initial vectors $\underline{x}(0)$ of the region R of Fig. 3.2 lead to "allowed" $\underline{x}(k)$ for all (continuous) values⁴ of k .

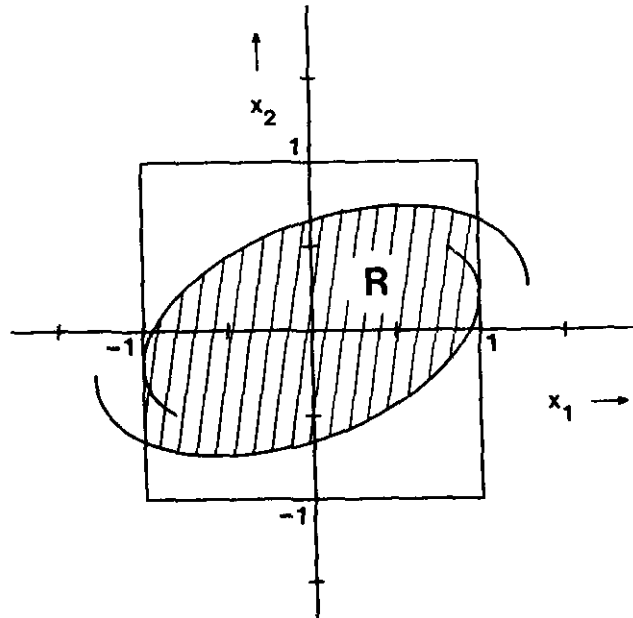


Fig. 3.2. Region R of initial states that never lead to overflow.

What occurs if $\underline{x}(0)$ is outside R ? Then, at some time instant k , the linearly determined $\underline{x}(k)$ might leave the unit square, and *overflow correction* has to be applied. This correction introduces one of two basic state modifications:

- (a) \underline{x} is moved towards the origin
- (b) \underline{x} is moved away from the origin.

⁴Note that the discrete character of k causes also some points outside R (but inside the square) to be allowed as initial conditions $\underline{x}(0)$, because parts of the continuous curves of Fig. 3.1. are not actually occupied.

Case (a) is wanted because it supports the natural linear motion; no oscillation occurs if all overflows are corrected this way. Case (b) is dangerous, because it compensates or even overcompensates the linear behaviour and, hence can (but need not) lead to oscillations. Of course, these statements ask for an unambiguous definition of "distance from the origin". Instead of the widely used euclidean norm our definition is guided by the linear state motion, according to (3.2).

Following

$$\underline{x} = X(\underline{e}_r \cos \varphi - \underline{e}_i \sin \varphi) \quad (3.3)$$

two variables X, φ can be associated with each state \underline{x} . Particularly, the variable X is determined from \underline{x} as

$$X^2 = \left[\frac{\underline{e}_r^t \underline{x}}{\underline{e}_r^t \underline{e}_r} \right]^2 + \left[\frac{\underline{e}_i^t \underline{x}}{\underline{e}_i^t \underline{e}_i} \right]^2. \quad (3.4)$$

Comparing (3.3) with the linear motion as described by (3.2) one recognizes $X = \tilde{X} e^{\Gamma k}$, i.e. a monotonically decreasing function $X(k)$. Combined with the fact that X^2 is a quadratic form in x_1, x_2 as formulated by (3.4), the parameter X^2 is a natural candidate for a Lyapounov function^{*}. Observe that the curves $X = \text{const}$ constitute a family of "concentric" ellipses (with axes along \underline{e}_r and \underline{e}_i) and that low- X ellipses are enclosed by high- X ellipses. Naturally, we choose X as the "distance from the origin".

Overflow correction is now visualized in Fig. 3.3. An uncorrected state point B is transformed into B', B'', B''' after applying saturation, zeroing, and two's complement, respectively. For this example all types lead to an increase of X and, hence, to a movement away from the origin. On the other hand, for point C this is only true for zeroing and two's complement.

^{*}Other Lyapounov functions are discussed under the head "limit cycles", cf. Section IV.

For some ellipse geometries it is possible to use appropriate overflow characteristics such that the state *always* moves towards the origin and *oscillations are suppressed*. Obviously this is not the case for the arbitrarily oriented ellipse of Fig. 3.3. However, it is easily recognized that for an ellipse whose axes coincide with the $x_1 - x_2$ -axes, each of the three overflow corrections satisfies the stability condition, while for an ellipse with a 45° inclination stabilization can be obtained at least with a saturation characteristic.

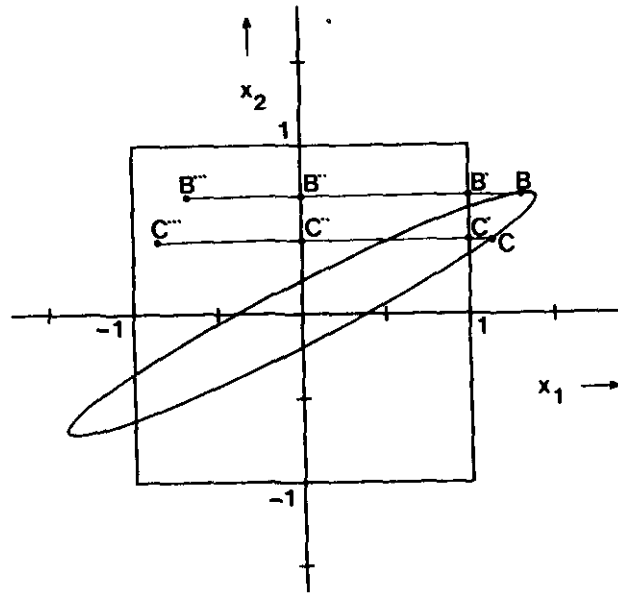


Fig. 3.3. Ellipse $X = \text{constant}$ in the phase plane.

It should be noted that in this picture the potentiality for stabilizing overflow is determined by the eigenvectors of A and not by the eigenvalues. While the latter determine the speed with which the trajectories are traversed, the eigenvectors determine the appearance of the ellipse, i.e. the orientation of the axes and their length ratio. These parameters are essentially determined by the *filter structure*, examples of which are the (a) normal filter (b) wave digital filter (c) direct-form filter as depicted in Fig. 4.1. The corresponding system matrices are [42], [36], [37]

$$A = \frac{1}{2} \begin{bmatrix} a & \sqrt{-4b-a^2} \\ -\sqrt{-4b-a^2} & a \end{bmatrix}, \quad A = \frac{1}{2} \begin{bmatrix} a+b+1 & a+b-1 \\ a-b+1 & a-b-1 \end{bmatrix}, \quad A = \begin{bmatrix} a & b \\ 1 & 0 \end{bmatrix} \quad (3.5)$$

(a) (b) (c)

where $a = 2e^{\Gamma} \cos \Omega$, $b = -e^{2\Gamma}$ are the coefficients of the characteristic polynomial.

For (a) the ellipse degenerates into a circle, so that all overflow characteristics lead to stabilization. For (b) and high Q -values ($b \rightarrow -1$) the ellipse axes coincide with the $x_1 - x_2$ -axes so that again all characteristics satisfy, while for (c) and high Q -values the ellipse axes have a 45° inclination so that stability is obtained with saturation. However, also for low Q -values and even for real poles stability can be guaranteed* [21],[22],[38].

Normal filters and wave-digital filters of orders higher than two can likewise be stabilized with all types of overflow characteristics [39]-[44]. However, higher-order direct-form filters are in general unstable with respect to overflow; high-period and chaotic oscillations have been observed in such structures [45]-[51], occasionally with oscillator applications in mind [52],[53]. Observe that every stability requirement yields sufficient conditions; often these conditions can be weakened with various analytic measures [54],[55] or with computer-generated Lyapounov functions [56],[57]. Attempts have also been reported with unconventional overflow characteristics [21],[58] and overflow signalling schemes [59]-[61]. Special investigations have been published on the stability properties of wave-digital filters [70]-[74], normal filters [75], lattice filters [76]-[79], block-state realizations [80]-[81], and multi-input-multi-output structures [82]-[83], while experimental results have been reported in [7]. Parasitic oscillations in more complicated systems, particularly those formed by single-input-single-output systems under looped conditions have been discussed in [47],[51],[84].

B. Forced-response stability

In the previous subsection we have discussed sufficient conditions guaranteeing that no zero-input overflow oscillations occur. The non-existence of such oscillations was viewed as an absolute design requirement that every usable filter has to meet.

*The pertinent proofs are constructed with other Lyapounov functions and other ellipse geometries, cf. eq. (4.8) of Section IV.

An ill-designed filter can exhibit autonomous oscillations under suitable initial conditions. Physically, these are e.g. determined through connecting the digital circuit to a power supply or as a residue of former (meanwhile terminated) input signals. Such an initial condition need not immediately cause overflow but can lead to it after a number of time steps. Thereafter overflow becomes periodic or asymptotically periodic or irregular (chaotic). All these instabilities are characterized by the non-existence of a time instant, after which overflow ceases to occur.

On the other hand, stability implies that such a time instant *does* exist. This requirement is also the starting point for the forced response stability to be discussed in this subsection [85]-[90].

Occasional overflows are allowed, but there has to exist a last overflow, after which the system behaves linearly and thus recovers from potential former overflows. Asymptotically ($k \rightarrow \infty$) there remains the "forced response", which is independent of the initial conditions and, as such, not affected by all former overflows.

Stability in this sense depends upon the excitation. For each digital filter a (possibly empty) set of input signals exists for which stability holds. An apparent minimum requirement is that only such input signals $u(k)$ are admitted for which the associated linear filter (without overflow correction) does not exceed the overflow level after some time k_0 . The ensemble of all such signals (with k_0 unspecified) is said to form the class U_0 (definition "A"). Besides this definition "A" an alternative definition "B" is in current use which examines $u(k)$ only for $k \geq k_0$. Following "B" we have $u(k) \in U_0$ iff there exists an initial condition at $k = k_0$ such that the linear filter does not exceed the overflow level for all $k \geq k_0$. Apparently, the past history of $u(k)$ in the "A" interpretation is condensed in the initial condition⁷ according to "B" so that the "tails" of the "A" signals form the class U_0 in the "B" sense [86]-[87].

⁷In an uncontrollable system it can occur that not all initial conditions can be generated with the aid of suitable input signals. In such an exceptional case, the "B" definition is more general. This definition was already introduced in Section II.

A stable filter with overflow correction always exhibits a *finite* number of overflows (which may be zero or one in special cases) after $k = k_0$, which number depends upon the initial condition at k_0 . Assuming that $u(k) \in U_0$, there is at least one initial condition (mostly a set of neighbouring initial conditions) with no overflow after $k = k_0$.

If stability in the above sense holds for all $u(k) \in U_0$, the filter is called "forced-response stable" with respect to U_0 [87]. Since excitations $u(k) \notin U_0$ are meaningless in the context of stability, the addition "with respect to U_0 " is often omitted. Weaker forms of stability are found with respect to subsets of U_0 such as U_0^c with a scale factor c satisfying $0 \leq c < 1$. Compared with U_0 the signal amplitudes are reduced by a factor c such that $u(k)/c \in U_0$ [91].

In this notation $c = 0$ corresponds to "zero-input stability" being the weakest form of stability. It is somewhat surprising that systems whose stability is guaranteed only for zero input also behave stable for most excitations of practical importance. In fact, only periodic or almost-periodic* signals appear to be able to produce forced-response instabilities (with commensurate periods) in such systems.

Concerning the analytic investigations of forced-response stability it is a lucky circumstance that the nonzero-input problem can be transformed into a zero-input problem with time-varying nonlinearities [87]. Let $\underline{x}(k)$ and $\underline{z}(k)$ denote the state vectors of the actual and the idealized filter with excitation $u(k)$ such that (cf. (3.1))

$$\underline{x}(k+1) = F\{A \underline{x}(k) + \underline{b} u(k)\}$$

$$\underline{z}(k+1) = A \underline{z}(k) + \underline{b} u(k)$$

then the difference vector $\underline{d} = \underline{x} - \underline{z}$ satisfies the *homogeneous* difference equation

$$\underline{d}(k+1) = F\{\underline{z}(k+1) + A \underline{d}(k)\} - \underline{z}(k+1). \quad (3.6)$$

*As an example, we refer to [92]-[94], where an "irrationally" sampled continuous-time sinusoid has evoked instability.

Let us consider a certain component of $\underline{x}(k+1)$ and $\underline{A_d}(k)$ and denote it provisionally by v and δ , respectively. Then the same component of the right-hand term of (3.6) reads as $F\{v+\delta\}-v$, i.e. a time-varying (due to $v=v(k)$) non-linear function of δ . With a linearly determined $v(k)$ the function $F\{v+\cdot\}-v$ is a shifted replica of $F\{\cdot\}$, with equal horizontal and vertical v -shifts of the F -plot. Fig. 3.4 shows the result for the three basic overflow characteristics.

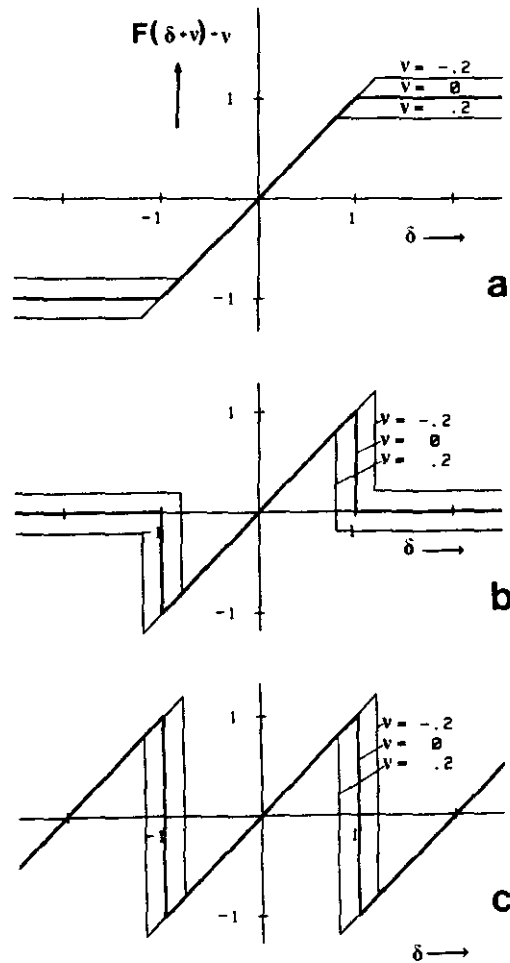


Fig. 3.4. Plots of $F\{v+\delta\}-v$ for (a) saturation, (b) zeroing (c) two's complement.

With the knowledge that for many structures (e.g. normal and wave-digital filters) the condition $|F(\delta)| \leq |\delta|$ ensures zero-input stability we can likewise conclude that (3.6) has a stable solution (with $\underline{d}(k) \rightarrow 0$ for $k \rightarrow \infty$) if $|F(v+\delta) - v| \leq |\delta|$. From Fig. 3.4 we conclude that this is true for saturation if $|v| < 1$, for zeroing if $|v| < 0.5$, and for two's complement if $v = 0$, i.e. for excitations that are elements of U_0 , $U_0^{0.5}$, U_0^0 , respectively, (in the sense of definition "A" as given above).

We conclude this section with some phenomena occurring in an *unstable* filter. For a given $u(k) \in U_0$ there exists a set of initial conditions, for which no overflow occurs. In general, there exists another set of initial conditions, which leads to a finite, nonzero number of overflows. Finally, due to the assumed instability, a third set of initial conditions gives rise to an infinite number of overflows. It is only in this situation that the instability becomes manifest. For a periodic excitation, the response, too, becomes asymptotically periodic, but the period need not be the same. Subharmonics can occur, but also completely different periods are observed [85]. In general, the asymptotic response is not unique, even if the periods of excitation and response are equal. Additional pulse excitations can lead to jump phenomena from one response to another [95],[96].

IV. Quantization limit cycles

Besides the large-amplitude overflow oscillations treated in the previous section, still other parasitic oscillations are observed in *recursive* digital structures, which have their origin in the quantization fine structure and, as a result, have relatively small amplitudes. These oscillations can occur under zero- or (nonzero)constant-input conditions and are generally called "*limit cycles*". Together with quantization noise (cf. Section V) they are considered as the most serious deviation from linear behaviour under normal operating conditions of a digital filter. In contrast with quantization noise, they can, however, be completely avoided. Unfortunately the involved techniques complicate the noise analysis such that a systematic noise minimization cannot be achieved with analytic tools. Thus in current literature we observe almost independent studies of limit cycle suppression and of noise optimization. The first problem mostly deals with quantization by magnitude truncation MT (or related methods) while the second is completely based on rounding R.

The main factor determining the occurrence of limit cycles is the quantization characteristic. In this section we mainly consider R and MT quantization; modified quantizations like controlled rounding CR and stochastic quantization require additional signals and, as such, more complicated descriptions than a simple characteristic.

A. Limit cycle suppression with Lyapounov and other deterministic methods

The analytical treatment of limit cycles resembles that of overflow oscillations. This implies an organization of the present section similar to that of Section III. Again we begin with second-order systems with complex poles under zero-input conditions, for which

$$\underline{x}(k+1) = f(A\underline{x}(k)) \quad (4.1)$$

likewise applies with the only modification that now $f(\cdot)$ is allowed to be a more general nonlinear vector function. The former strictly component-wise application of the scalar overflow characteristic $F(\cdot)$ is thus abandoned.

This generalization allows for the most general quantization scheme, in which not only state variables (= components of \underline{x}) are subject to quantization, but also intermediate products or sums.

Whereas in the overflow problem \underline{x} denotes a continuous set of variables, quantization implies a discrete-amplitude character of \underline{x} with all x_i integer multiples of the quantum q . Reckoning with the fact that all solutions of the homogeneous equation (4.1) are bounded for $k \rightarrow \infty$, any filter with quantized state variables can consequently be viewed as a *finite-state machine*.

While for any arbitrary initial condition $\underline{x}(0) \neq \underline{0}$ the state $\underline{x}(k)$ in a *linear filter* asymptotically approaches the origin ($\underline{x}(k) \rightarrow \underline{0}$ for $k \rightarrow \infty$), this is not the rule for the nonlinear filter described by (4.1). Instead, for some $k = k_0$ the state $\underline{x}(k)$ enters a limit cycle. This is a periodic motion characterized by N state points which are cyclically occupied by $\underline{x}(k)$. "Accessible" limit cycles can be entered from points outside the cycle which together with all their predecessors form a (mostly immense) set of state points to be assigned to such a cycle [97]-[99]. On the other hand, "inaccessible" cycles have to be started on the cycle itself. Limit cycles of period 1 consist of one point, which can be accessible or inaccessible. If and only if the origin $\underline{x} = \underline{0}$ is ultimately reached from any initial condition (implying accessibility of the origin) the filter is *limit-cycle free*.

Without any quantization (corresponding to the ideal, linear filter), the trajectory of the state vector $\underline{x}(k)$ would follow an *ellipse-like curve* spiraling towards the origin, as shown in Fig.3.1. In the actual filter, quantization introduces a slight modification of the state vector such that its quantized version becomes a point in the quantization grid, located in the close vicinity of the state before quantization. Like the overflow correction discussed in Section III, quantization can be associated with a state motion towards the origin or away from it. The first motion supports the linear motion and ensures *freedom of limit cycles* if quantization is always performed this way. Clearly, this rule provides a *sufficient* condition. Conversely, quantization correction away from the origin does not admit any conclusion: limit cycles can, but need not occur.

The above statements ask for a definition of "distance from the origin". In contrast with the straightforward definition of Section III we take a more general standpoint by identifying any "Lyapounov energy function" of the associated linear filter with the square of the distance from the origin. Such a function $P(\underline{x})$ is

- (a) a quadratic form* $P = \underline{x}^T Q \underline{x}$, where
- (b) Q is symmetrical and positive definite, corresponding to $P > 0$ for all $\underline{x} \neq \underline{0}$, and
- (c) the system dynamics is such that P decreases with increasing time:
 $P(\underline{x}(k+1)) < P(\underline{x}(k))$.

For the linear system with $\underline{x}(k+1) = A \underline{x}(k)$ condition (c) reads as

(c') $Q - A^T Q A$ is positive definite.

All matrices Q satisfying conditions (b) and (c') are candidates for "energy matrices" defining an appropriate energy function. If, for one of such matrices, quantization lowers the energy P , freedom of limit cycles is guaranteed. In terms of (4.1) this condition reads

$$f^T(\underline{x}) Q f(\underline{x}) - \underline{x}^T Q \underline{x} < 0 \quad \forall \underline{x} \quad (4.2)$$

If (4.2) holds, condition (c) for the energy function is satisfied in the linear and in the nonlinear filter, so that P is also a Lyapounov function of the nonlinear system. Care must be taken if $<$ is replaced by $=$ in (4.2) so that energy is not changed by quantization. If, moreover, "definite" in condition (c') is relaxed into "semi-definite", it can occur that energy remains constant, associated with the risk of a limit cycle. Such a situation occurs for a marginal choice of Q for which in the linear filter periods of low and high energy decrease alternate. The other extreme is a continuous decrease in exponential form, as found for the distance definition of Section III.

Historically, Lyapounov theory (with appropriate modifications) was first applied to wave digital filters, i.e. structures derived from classical LC-twoports. For the second-order section of Fig. 4.1b with A given in (3.5b), Q is advantageously chosen in the diagonal form

*Notice that $P = \text{const}$ represents an ellipse in an x_1 - x_2 -plane.

$$Q = \frac{1}{2} \begin{bmatrix} 1+a-b & 0 \\ 0 & 1-a-b \end{bmatrix}. \quad (4.3)$$

With this choice the ellipses $P = \text{const}$ have axes parallel to the coordinate axes. Further, the linear energy decrease per time step reads

$$P(\underline{x}(k+1)) - P(\underline{x}(k)) = -(1-a-b)(1+a-b)(1+b)[x_1(k)+x_2(k)]^2/4 \leq 0. \quad (4.4)$$

Observe that P is a marginal Lyapounov function, since for $x_1 + x_2 = 0$ the energy P remains constant. It appears, however, that one time step after this occurs, $x_1 + x_2 \neq 0$ so that energy again decreases. Applying MT quantization on the individual state variables reduces $|x_1|$ and $|x_2|$ and, consequently, also P , so that limit cycles are forbidden [70]-[73],[100],[101].

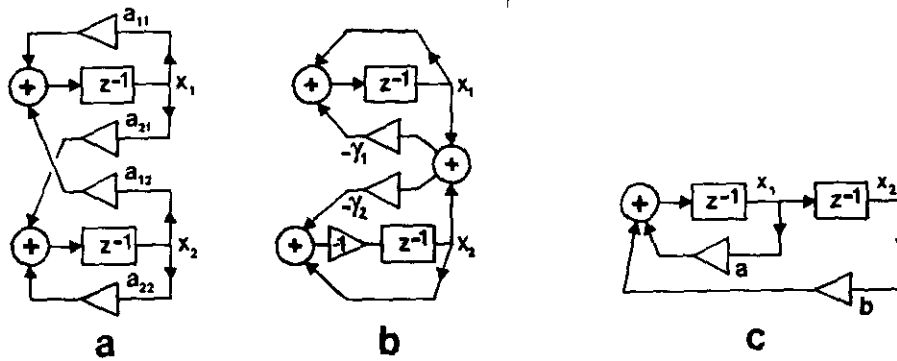


Fig 4.1. Recursive parts of second-order structures in normal form (a), wave digital form (b) and direct form (c).

The filter coefficients are interrelated by $a_{11} = a_{22} = a/2$;

$$a_{12} = -a_{21} = \frac{1}{2} \sqrt{-4b-a^2}; \quad y_1 = \frac{1}{2}(1-a-b), \quad y_2 = \frac{1}{2}(1+a-b).$$

A widely used, straightforward design of a second-order section is in the *direct form* of Fig. 4.1c with A given by (3.5c). Here we choose advantageously

$$Q = \begin{bmatrix} 1-b & -a \\ -a & 1-b \end{bmatrix} \quad (4.5)$$

with ellipses $P = \text{const}$ under 45° . The linear energy decrease per time step is given by

$$P(\underline{x}(k+1)) - P(\underline{x}(k)) = -(1+b)[x_1(k+1) - x_1(k-1)]^2 \leq 0. \quad (4.6)$$

Although, due to $x_2(k+1) = x_1(k)$, only the state variable x_1 needs to be quantized, MT is unable to perform that task throughout without energy increase. This can be understood geometrically, since in parts of the ellipse MT causes a state motion away from the origin¹⁰, cf. Fig. 4.2.

In an alternative approach, we combine the linear and nonlinear (quantization) operation, which at least yields a non-increasing energy function. First recall that $x_2(k+1) = x_1(k)$ so that some grid point M in the state space is always linearly transformed into a point on a straight line through the mirror point M^* with respect to the 45° line, cf. Fig. 4.2. Let M' denote the result of this linear transformation of M , and let M'' denote the result of the subsequent quantization, then M' lies on the line segment $x_2(k+1) = x_1(k)$ inside or on the Lyapounov ellipse (due to (4.6)) while M'' is desired to lie there, too.

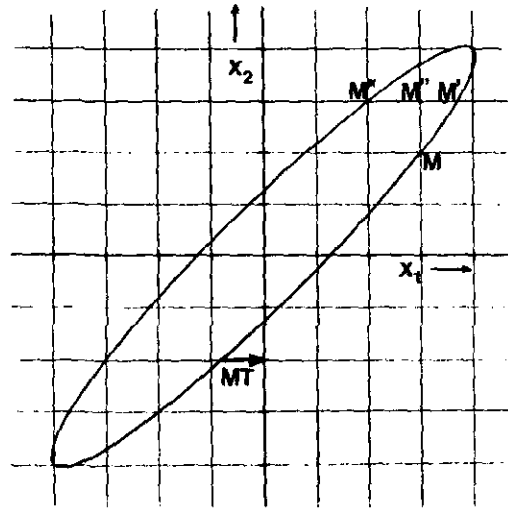


Fig. 4.2. State-space of a second-order direct form filter.

¹⁰Other (permitted) choices of Q -matrices might lead to such ellipses, that MT throughout reduces energy, cf. (4.8).

Realizing that, according to their definitions, M'' and M^* are grid points, we conclude that M^* is a suitable candidate for M'' , but also any (if existent) intermediate grid point between M' and M^* . To achieve minimum error, we choose for M'' the grid point nearest to M' . This construction yields the quantization rule that $x_1(k+1)$ has to be quantized "in the direction" of $x_2(k) = x_1(k-1)$.

Unfortunately, this "controlled rounding" CR does admit constant-energy limit cycles of periods 1 or 2, in which the state jumps between M and M^* or, for $M=M^*$, remains constant at M [102].

It is an advantage of CR that not only zero-input stability (with the exceptions mentioned) is achieved, but also stability under any *constant-input condition*, because the quantization rule only involves signal differences [102]. With some additional hardware, limit cycles of periods 1 and 2 can be suppressed, too, but at the expense of the general constant-input stability [103],[104]. This is not the case for CR applied to wave digital filters, in which stability holds for all constant inputs [105]-[108]. A number of other solutions are based upon the elementary insight that CR can be reduced to MT by first subtracting the control signal, then applying MT, and finally again adding the control signal. Structures thus derived from wave digital sections are often presented in multi-output form with lowpass, highpass, bandpass and allpass outputs [109]-[113]. Also second-order structures of the wave-digital type with only one MT quantizer have been devised, which are stable for zero-, constant-, and alternating input signals [114]-[116]. In these solutions, not the states are quantized, but some intermediate signal¹¹.

A third important class of second-order sections are the *normal* filters (also "coupled sections") cf. Fig. 4.1a. In a certain sense, they exhibit the best overflow and limit cycle behaviour (and, moreover, excel with respect to quantization noise), a benefit that has to be paid for with twice the number of multiplications (4 instead of 2 for wave digital and direct-form sections).

The A matrix is given by (3.5a), the Q matrix is the unit matrix, and P is the square of the euclidean distance from the origin. The energy decrease per unit time reads

¹¹This is an example of a more general vector function $f(\bullet)$ in (4.1).

$$P(\underline{x}(k+1)) - P(\underline{x}(k)) = -(1+b)(x_1^2(k) + x_2^2(k)) \leq 0 \quad (4.7)$$

and vanishes only for $\underline{x}(k) = \underline{0}$. Thus, P is a regular Lyapounov function, whereas the corresponding functions for wave digital and direct-form filters only belong marginally to this class (in a strict sense, they are not Lyapounov functions). The curves $P = \text{constant}$ now degenerate into concentric circles, and MT applied to the individual state variables ensures zero-input stability.

It is a common property of normal and wave digital filters that Q is a diagonal matrix and that $P = \text{constant}$ are ellipses oriented parallel to the coordinate axes. Only *this* ellipse geometry allows for *MT quantizations* applied to the individual state variables, without risk of limit cycles [117]-[122]. The question arises: which A matrices admit a diagonal "energy matrix" Q ? This problem has the solution [42], [43]

$$|a_{11} - a_{22}| + \det A < 1. \quad (4.8)$$

All filters with matrices A satisfying (4.8) remain zero-input stable under MT quantization of the individual state variables and, for the same reason, under any overflow correction. Do the three basic section types satisfy (4.8)? The answer is "yes" for the normal form, "yes" iff $|a| - b < 1$ for the direct form, "almost yes" for the wave digital form with the inequality sign replaced by an equality sign (reflecting the marginal character of the Lyapounov function). Also "lattice filters" and "minimum-norm filters" [76], [44] satisfy (4.8).

Higher-order filters with orders $n > 2$ are often designed through appropriate generalizations of second-order sections. In the present state of the art, only the ladder filters, particularly the wave digital filters (WDF) appear to be sufficiently developed such that a direct n -th order approach is feasible. Due to the availability of a recent review article [36] on WDF design, including nonlinear parasitic effects, we can confine ourselves to the brief statement that, because of its inherent passivity properties, MT can often be (partly) replaced by R without risk of limit cycles, which results in lower quantization noise levels [123], [124].

In addition, investigations on limit cycles in floating-point arithmetic WDF design [125],[126] and in half-synchronic filters [74] as well as filters under looped conditions [84] deserve to be mentioned. Apart from the WDF approach, most higher-order filters are designed in parallel- or cascade form, in which cases knowledge about stability of second-order sections suffices.

Concerning stability with respect to constant (nonzero) inputs, which has so far been touched upon only in ad hoc situations, we mention a general principle to convert a stable *autonomous* system into a system with *input terminals* stable under any constant excitation. In more explicit terms, let an autonomous system satisfy (4.1) (where f has the original meaning of a scalar quantization characteristic) and let further the solution of (4.1) approach zero for $k \rightarrow \infty$ (expressing freedom of limit cycles), then through suitably supplying such a system with an input terminal, a constant-input stable system can be created as follows. At each quantization point i , some signal v_i is *added* after quantization, while the same signal is *subtracted* after the sum signal has passed the subsequent delay element. The pair of injected signals v_i is proportional to the input signal, $v_i(k) = b_i u(k)$, so that the state in the modified system satisfies

$$\underline{x}(k+1) = f\{A(\underline{x}(k) - \underline{b} u(k))\} + \underline{b} u(k). \quad (4.9)$$

For a *constant* excitation $u(k) = U$ the difference vector $\underline{x}(k) - \underline{b}U$ satisfies the original equation (4.1) so that the state vector asymptotically approaches the stationary solution $\underline{b}U$ without superimposed limit cycles [127]-[134].

So far, all limit-cycle suppressing mechanisms made use of MT (including the related CR) quantization, utilizing its energy reducing property. On the other hand, *rounding* R can amplify the signal magnitude by a factor $c \leq 2$, where the maximum factor $c = 2$ occurs for a signal magnitude equal to half a quantization step. In an attempt to achieve freedom of limit cycles also for R quantization, the nonlinear energy increase has to be compensated by an equal energy decrease associated with the linear filter operation. In concrete terms, the necessary damping finds expression in the condition $\|A\| < \frac{1}{2}$, where $\|A\|$ denotes the norm of the system matrix.

If this condition is not met by the design requirements, the matrix A can be transformed into some power A^L by means of a "block-state realization" or a "matrix-power feed-back" such that $\|A^L\| < \frac{1}{2}$ and R quantization can be applied without risk of limit cycles [80]-[83]. For high- Q filters this method, however, requires a great amount of hardware.

Besides the basic limit-cycle suppressing concepts discussed so far, a vast amount of ideas has been published dealing with special structures and more complicated (deterministic) stabilization methods. Due to space limitation, we can only present them in summarized form. Multirate filters [135], error feedback [136]-[138], digital incremental computers [139] and other special structures [140]-[143] belong to this category. Much attention has been paid to the stability of direct-form second-order sections in certain regions of the a - b -parameter plane and with respect to certain cycle periods (particularly periods 1 and 2). Most of the pertinent publications belong to the earlier period of research on nonlinear effects in recursive digital filters; as such, they have essentially contributed to our present understanding of these phenomena. With the advent of modern universal (i.e. for all coefficients and all periods) methods for limit cycle elimination the results of these investigations have to some extent lost their practical value [144]-[149]. In this context, frequency-domain criteria formed a powerful analytic tool to derive sufficient stability criteria [150]-[154]. Special investigations concern coupled-form filters [155], cascaded sections [156]-[158], sections with non-uniform internal wordlength [159],[160] and with small input signals [161].

The instabilities mentioned at the end of Section III have their counterparts also in the context of quantization effects. Under *periodic excitation* the solutions likewise need not be unique; jump phenomena and *subharmonics* (with relatively small amplitudes) result from such instabilities [162]-[167]. Measures for the suppression of such subharmonics have been proposed [168]-[170].

B. Limit cycle suppression with stochastic methods

Another way to eliminate the various types of limit cycles is to control quantization through an external random signal. This way potential conditions favourable to the occurrence of parasitic oscillations are irregularly disturbed which results in an asymptotic, albeit noisy approach of the zero state.

The disadvantages of such stochastic methods are evident: they require additional random sources (preferably independent sources for all quantizers) and, at a first glance, yield additional quantization noise. The latter point is, however, compensated by the flatness of the noise spectrum that contrasts with the (mostly) narrow bandwidth of the noise generated by MT quantization. Particularly in high-Q filters the ultimate noise contributions at the output terminals can thus be considerably smaller than those occurring with deterministic stabilization methods. Another advantage is the avoidance of cross-talk, as discussed in Section II.

The simplest method is *random rounding*, where the decision about the handling of the least significant bit is exclusively left to the exterior random signal [171],[172]. A variation on this strategy is found when the unquantized signal is supplemented with *random dither*, whose spectral distribution is flat and whose amplitude distribution is uniform in the interval $[-q/2, q/2]$. The complete signal is subsequently subject to R quantization [173],[174]. In contrast with this "uniform random dither", the former random rounding is occasionally referred to as "binary random dither". In rough terms, uniform r.d. has a better noise performance, whereas binary r.d. is superior with respect to limit cycle suppression. In a variant, uniform r.d. is subject to spectral shaping, particularly with a bandstop characteristic. The resulting "*bandstop dither*" has an improved noise performance, to be sure, but is costly to implement [175],[176].

Guided by the inherent properties of R and MT quantization, one can combine their respective merits into "*random quantization*", in which an external generator randomly switches between R and MT quantization, with comparatively short MT operating times. This way the excellent R noise properties are coupled with the stabilizing capability of MT quantization, which has to be paid for with a prolonged limit cycle expiration time [177]-[180].

C. Properties of limit cycles

So far attention was focused on the suppression of all forms of limit cycles. Occasionally, however, such parasitic oscillations are wanted (as in a digital oscillator) or at least tolerated (with the argument of saving additional hardware). In either case, it is worthwhile to get acquainted with *various properties* of limit cycles, if these can occur. This is particularly true for their *maximum amplitude*, as its knowledge enables a user to apply a coarse re-quantization such that a potential limit cycle remains smaller than a quantization step. This "screening" of limit cycles is, in fact, the classical technique to cope with this phenomenon [181]. Usually, the requantization is only applied at the lowest signal levels (thus forming a threshold detector), although it then fails to work under constant-(nonzero) input conditions. In view of the modern suppression methods the "screening" method introduces a relatively high degree of signal distortion.

In second-order sections with complex poles at $z = e^{\Gamma \pm j\Omega}$ (with $\Gamma < 0$, $\Omega > 0$) the limit cycle amplitude is tightly bounded by the expression

$$q(\sin\Omega)^{-1} \sum_{k=0}^{\infty} e^{\Gamma k} |\sin(k+1)\Omega|$$

being the ℓ_1 norm $\|h\|_1$ of the filter's impulse response $h(k)$ (apart from the factor q). To the best of our knowledge, $\|h\|_1$ cannot, in general, be cast in closed form; this fact has motivated many authors to derive simpler upper bounds, more or less higher than $\|h\|_1$ [182]-[200].

Besides the amplitude, also the power of a limit cycle has an upper bound [201]-[203]; the same holds for certain norms of the output signal [204], [205]. Many papers deal with amplitude bounds in special structures, such as filters with error feed-back [206],[207], multirate filters [208], coupled-

form filters [209]-[212], digital incremental computers [213]-[216], wave digital filters with internal oscillations [217], cascade sections [218]-[220]. Special attention has been devoted to rolling-pin limit cycles [221],[222] and to filters using floating-point arithmetic [223]-[228]. Amplitude bounds of limit cycles have also been derived with Lyapounov functions [229]-[232] and computer simulations [233]-[237].

Many investigators have studied special forms of limit cycles. Besides the aforementioned rolling-pin [221],[222] almost sinusoidal waveforms have been explored or simply presupposed [144]-[150],[189],[190],[209]-[216],[229]-[232]. In many structures periods 1 and 2 attract particular attention [146],[147],[211]-[220],[240]-[242] as well as other special periods [162]-[167],[201],[202],[229]-[239]. Symmetry considerations and associated pairing of limit cycles can be found in various papers [97],[229],[243]. Finally we refer to some experimental observations [157],[158],[218],[219].

V. Quantization noise

A. Error statistics

In contrast to the largely deterministic treatment of the non-linear effects of quantization and overflow in a zero-input or a constant-input situation, the error during normal operation of a digital filter, i.e. in the presence of a more or less arbitrary input signal, is generally treated on a stochastic basis. The keyword here is 'arbitrary': in strict mathematical terms only a random signal, when passed through a quantizer, can cause a random error. In case of a rounding quantizer, however, even a deterministic, non-periodic input may result in a random nature of the rounding error ϵ_R , which is then designated *quantization noise* or *roundoff noise*.

If the input to a rounder has a range of sufficiently many quantization steps and a sufficient spectral width, the white-noise model for the rounding error (cf. Section II) may be applied [30]. Specifically, the error is uniformly distributed in the range $[-q/2, q/2]$ if the error function $\epsilon_R(x)$ of Fig. 2.1 operates on a large number of different values of the argument x . This will be the case for any non-periodic signal, the wordlength of which is reduced by a sufficient number of bits. As a result, the error carries a power of $q^2/12$, where q is the quantization step size. In most practical cases successive errors are uncorrelated with each other and with the quantized signal [245]. In order to determine the total power of this quantization noise (or the noise spectrum) at the filter output, we simply add the contributions of the various quantizers each filtered linearly to the output.

The statistics of the truncation error ϵ_{MT} are not that readily evaluated. Of course the quasi-rounding part ϵ_{QR} of this error (cf. Fig. 2.2) is subject to the above observations concerning ϵ_R . The sign part ϵ_{SGN} , however, introduces an error that is strictly deterministic, leading to ordinary distortion and crosstalk as was already stated in Section II. Nevertheless, a statistical treatment of this error is worthwhile if the signal to be quantized is random. In a current analysis the filter input signal is assumed to be a gaussian random process [246], [248], [249]. Again, the sign part ϵ_{SGN} of the truncation error is decomposed into a linear part (with an

appropriate negative slope) that is completely correlated with the quantized signal and an additive (uncorrelated) part that is interpreted as coloured noise. Figure 5.1 shows the quantization noise models for rounding and magnitude truncation, where s_{xx} denotes the autocorrelation function of any signal x .

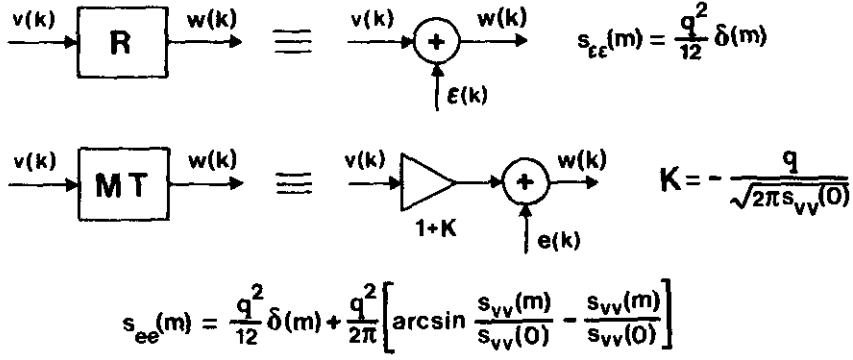


Fig. 5.1. Quantization noise models for R and MT. The model for the MT-error is valid for Gaussian excitation of the filter.

The multiplier $1+K$ causes a level-dependent (slight) modification of the filter coefficients and, ultimately, filter detuning, an effect which decreases with increasing signal amplitude. Moreover we observe that although the noise powers associated with the quasi-rounding part and the sign part of the truncation error are almost equal at their source, viz.

$$s_{ee}(0) = \frac{q^2}{12} + \frac{q^2}{2\pi} \left(\frac{1}{2\pi} - 1 \right) \approx \frac{q^2}{12} + \frac{q^2}{11}, \quad (5.1)$$

the contribution of ϵ_{SGN} to the output noise power may substantially exceed that of ϵ_{QR} . This is the case e.g. for an MT-quantizer in the feedback loop of a high-Q filter: The frequency region in which the source noise spectrum is concentrated coincides with the passband of the linear system from quantizer to filter output. The source noise spectrum exhibits a similar resonance peak, as dictated by the filter poles [255].

The statistics of the quantization error are contained in a number of papers [245]–[270], [30], [25]. These include *random rounding* [247] (also ref. [171], [172]), which can be modelled similar to conventional rounding

(R), and an error analysis of the *distributed arithmetic* [256] as well as *floating point arithmetic* [265]-[270], [25]. *Controlled rounding* (CR) can be translated into MT [264] leading to comparable noise powers [250].

B. Optimal structures

Most papers on quantization noise adopt the white-noise model (implying R quantization) and deal with fixed-point arithmetic with a certain quantization step size q . The resulting noise power at the filter output depends on

- the filter parameters and consequently on the desired frequency response,
- the scaling criterion which is used,
- the filter structure.

Starting from a given filter structure (e.g. direct form, normal form, wave digital structure) we can significantly reduce the influence of the filter parameters on the output quantization noise power. This is done by means of *error-feedback*, which we will discuss in the next subsection. Conversely, given a certain (linear) system function, we may try to find the optimal structure, i.e. the structure achieving the lowest possible output noise power. This problem was first solved using a state-space description of a digital filter [289], [296], [305], and a great number of papers has been devoted to optimal and sub-optimal structures since that time [271]-[314]. Furthermore, error-feedback and structure optimization have been compared as to their noise reduction performance, or been considered in a combined strategy in order to achieve even lower noise levels [315]-[328].

As for scaling, clearly, the more conservative we choose our scaling rule, the less efficiently we exploit the machine-representable state-space, resulting in poor S/N-ratios. Conversely, if we scale too moderately, the probability of overflow will be intolerably high. Scaling is performed by normalizing the impulse responses $f_i(k)$ from filter input to the various internal nodes (indexed i) with respect to their l_p -norm $\|f_i\|_p$ (such that $\|f_i\|_p = 1$, where p can be any positive integer) or, similarly, by normalizing the corresponding frequency responses $F_i(e^{j\Omega})$ with respect to their L_q -norm $\|F_i\|_q$ (such that $\|F_i\|_q = 1$) [35].

The most conservative is l_1 -scaling, by which absence of overflow is guaranteed for the total class of bounded input signals (i.e. $\{u(k)\} \leq 1$ or equivalently $\|u\|_\infty = 1$, where the overflow-level is set to unity). Next we have L_∞ -scaling which prevents overflow for L_1 -bounded input signals (which include unit-amplitude sinusoids). The most widely used is l_2 -scaling (which is identical to L_2 -scaling), primarily because it follows directly from the process of structure optimization. The following relations can be shown to hold [35].

$$\|f_i\|_2 = \left[\sum_{k=0}^{\infty} f_i^2(k) \right]^{1/2} \leq \|F_i\|_\infty = \max_{\Omega} |F_i(e^{j\Omega})| \leq \|f_i\|_1 = \sum_{k=0}^{\infty} |f_i(k)|. \quad (5.2)$$

For Gaussian random processes, we can interpret $\|f_i\|_2^{-1}$ as the scaling factor that ensures equal overflow probabilities for the i -th node register and the filter input register. Similarly, if we scale by setting $\delta \cdot \|f_i\|_2$ to unity, we can interpret δ as the number of standard deviations that can be represented in the i -th node register if the filter input is unit-variance white noise.

Before proceeding to a brief review on structure optimization, we point out that, historically, this was preceded by various strategies (heuristic as well as algorithmic using dynamic programming) to optimize a cascade connection of (in general) direct-form second-order sections. Higher-order filter design by combination of second-order sections is of interest in order to reduce the parameter sensitivity of a direct-form realization. Optimal structures, in the sense of low sensitivity and low noise, are achieved by a certain pole-zero pairing and ordering of the various sections, without changing their internal structure [350]-[366]. As a rule of thumb we may realize a complex conjugate pair of poles together with the nearest pair of zeros, working consecutively, starting with the highest-Q poles. A similar rule may be given for the correct section ordering [364].

Structure optimization (for a comprehensive treatment ref. [2]) is based on a state-variable description of a digital filter:

$$\underline{x}(k+1) = A \underline{x}(k) + \underline{b} u(k) \quad (5.3)$$

$$y(k) = \underline{c}^t \underline{x}(k) + d u(k) \quad (5.4)$$

A state-space filter is a direct realization of the state equations in which the components of the two (column) vectors \underline{b} and \underline{c} are the multipliers in the signal paths from the filter input ($u(k)$) to the inputs of the delay elements ($\underline{x}(k+1)$) and from the outputs of the delays ($\underline{x}(k)$) to the filter output ($y(k)$) respectively. The l_2 -scaling constraint $\delta \cdot \|f_i\|_2 = 1$ can be met if we apply a diagonal coordinate transformation D^{-1} to the state \underline{x} of a state-space filter to obtain a scaled state \underline{x}' :

$\underline{x}' = D^{-1} \cdot \underline{x}$, where $D = \text{diag} \{ \delta \cdot \|f_1\|_2, \delta \cdot \|f_2\|_2, \dots, \delta \cdot \|f_n\|_2 \}$, and n is the order of the filter. As a result the state transition matrix A undergoes a similarity transformation

$$A' = D^{-1} A D. \quad (5.5)$$

The trace and the determinant of a matrix (as well as its eigenvalues and hence the filter transfer function) are invariant under such a transformation. This is true for any non-singular transformation T^{-1} applied to the state \underline{x} . In fact, to find the optimal structure, we seek a coordinate transformation that minimizes the output noise power under the constraint that the filter remain properly scaled.

If we use one quantizer per state variable at the (double-precision) summation node preceding each delay element (cf. Fig. 4.1a for the second-order case), we can assume n rounding errors $\epsilon_1, \dots, \epsilon_n$ to be injected at these nodes, each carrying a power of $q^2/12$. Specifically, the total noise variance at the output of the scaled filter is given by

$$\sigma_{\text{total}}^2 = \frac{q^2}{12} \sum_{i=1}^n \|g_i'\|_2^2 = \frac{\delta^2 q^2}{12} \sum_{i=1}^n \|f_i'\|_2^2 \cdot \|g_i'\|_2^2 = \frac{\delta^2 q^2}{12} \sum_{i=1}^n \|f_i\|_2^2 \cdot \|g_i\|_2^2, \quad (5.6)$$

where the impulse responses $g_i'(k)$ describe the paths from the various error sources to the output of the scaled filter. The first equality states that the (uncorrelated) contributions to the total output noise may be added. The second and third equality are valid, because for the scaled filter we have $\delta \cdot \|f_i'\|_2 = 1$ and $g_i'(k) = \delta \cdot \|f_i'\|_2 \cdot g_i(k)$ since scaling affects f_i and g_i in a reciprocal manner, leaving their products unchanged. As a result, the order of the two operations 'scaling' and 'optimization' may be reversed;

instead of finding a non-singular transformation of the state \underline{x} which minimizes $\sum \|g_i\|_2^2$ under the constraint of proper scaling, we may optimize the unscaled filter, i.e. minimize $\sum \|f_i\|_2^2 \cdot \|g_i\|_2^2$ using an unconstrained transformation, and scale the resulting unscaled optimal filter in a final step, using the above diagonal transformation D^{-1} .

In order to facilitate the necessary calculations, two fundamental matrices K and W are introduced, which are positive definite and symmetric.

$$K = A K A^t + \underline{b} \underline{b}^t = \sum_{k=0}^{\infty} A^k \underline{b} \underline{b}^t (A^k)^t \quad (5.7)$$

$$W = A^t W A + \underline{c} \underline{c}^t = \sum_{k=0}^{\infty} (A^k)^t \underline{c} \underline{c}^t A^k \quad (5.8)$$

Note that the l_2 -norms squared of the impulse responses f_i and g_i are on the main diagonal of K and W resp., so $K_{ii} = \|f_i\|_2^2$ and $W_{ii} = \|g_i\|_2^2$. The implicit definitions of K and W allow these norms to be computed by solving two sets of linear equations. This is an appealing property since we now can scale the filter and calculate the output roundoff noise using only linear algebra not involving computation of infinite summations.

As it turns out, a necessary and sufficient condition for optimality of an n -th order filter structure is that the matrices K and W are directly proportional after scaling, i.e. $W' = \rho^2 \delta^4 \cdot K'$. Since K' has equal elements on its main diagonal ($K'_{ii} = \delta^{-2}$), so must have W' . This means that the optimal filter is characterized by that the various noise sources at the quantizers contribute equally to the total output noise. The factor ρ^2 is called the noisegain of the optimal filter, a quantity which depends only upon the filter transfer function. The trace of W' ($\text{tr}(W') = n \cdot \rho^2 \delta^2$) is the sum of eigenvalues of W' and, in view of the above relation, $\rho^2 \delta^2$ times the sum of square roots of eigenvalues of the product matrix $K' W'$. The eigenvalues of KW are invariant under a coordinate transformation ($K \rightarrow T^{-1} K T^{-t}$, $W \rightarrow T^t W T$ and $K W \rightarrow T^{-1} (K W) T$) and so we can calculate the noisegain and the output noise of the optimal filter without actually performing the optimization (which will be the case if we are only interested in what can be achieved):

$$\min \sigma_{\text{total}}^2 = n \cdot \frac{\delta^2 q^2}{12} \cdot \left[\frac{1}{n} \cdot \sum_{i=1}^n \mu_i \right]^2, \quad (5.9)$$

where μ_i are the square roots of eigenvalues of KW , the so-called second-order modes of a filter. Hence, the noisegain of the optimal filter is equal to the arithmetic mean squared of the second-order modes of the filter.

In the case of second-order filters ($n=2$) the condition $W' = \rho^2 \delta^4 K'$ can also be written as $W' = \rho^2 \delta^4 M K' M$, where $M = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ i.e. a permutation matrix [296].

This condition is met if we choose $A'^t = M A' M$ and $\underline{c}' = \rho \delta^2 M \underline{b}'$ in (5.8). This means that a second-order optimal filter is characterized by¹² [296]

$$a_{11} = a_{22} \quad (5.10)$$

$$b_1 \cdot c_1 = b_2 \cdot c_2, \quad (5.11)$$

where a_{ij} , b_i , c_i are the elements of the state-transition matrix A , the input-state vector \underline{b} and the state-output vector \underline{c} , respectively. In each second-order state-space filter we are free to choose the parameters a_{ij} , b_i and c_i to satisfy the above relations and will still be able to realize any second-order transfer function [282]. The optimal filter is obtained after scaling in a final step using $D = \delta \cdot \text{diag} \{ \sqrt{K_{11}}, \sqrt{K_{22}} \}$.

Again, higher-order filter design is based on combining second-order stages (in cascade or in parallel) but now this has a practical reason: The number of multiplications per output sample for an n -th order, minimum noise filter is $(n+1)^2$, whereas a cascade of second-order subfilters (of the state variable type) uses only $4n+1$ multipliers. The price we pay, of course, is a somewhat inferior performance of our design.¹³ A connection of second-order sections optimized in isolation is called a sectional optimal structure. If we optimize a connection of second-order state-variable filters as a whole, the overall filter is called block optimal.

¹²The primes are omitted since both (5.10) and (5.11) are unaffected by scaling.

¹³Contrast this with direct-form filter design, where the cascade of second-order stages is highly superior to the n -th order design.

C. Error-feedback and related noise reduction strategies

Error or residue feedback [315]-[349] is a noise reduction strategy that makes use of the discarded bits of a quantization operation. The basic idea is to introduce zeros in the path from the error source to the filter output to compensate for the poles that tend to blow up the quantization noise, especially for narrow-band filters. Error-feedback leaves the filter response unchanged but alters the output noise spectrum, hence it is also designated *error spectrum shaping* (ESS). Most papers on ESS deal with the second-order direct form with first- or second-order error-feedback (the distinction depending on whether one or two past samples of the quantization error are retained and processed). The basic structure is shown in Fig. 5.2, where the quantization error $\epsilon(k)$ is understood to be $y(k)-v(k)$.

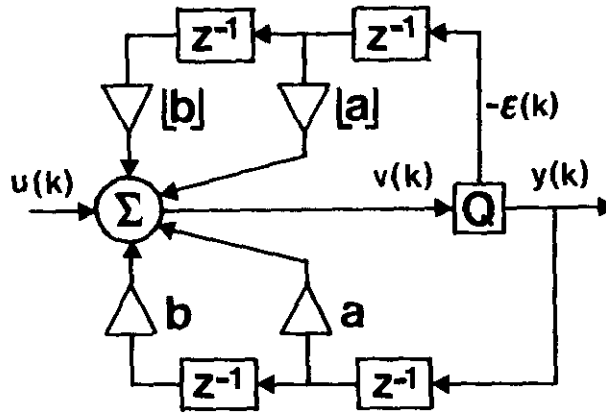


Fig. 5.2. Basic error-feedback structure.

The summation operates on double-precision numbers. The multipliers $[a]$ and $[b]$ have integer values nearest to a and b , resp., and as a result give rise to very simple additional hardware (shifters or inverters). Moreover, no extra quantization is required in the feedback loop of the error ϵ , which is a pseudo-double-precision number.

We can assume an error $\epsilon(k) = [a]\epsilon(k-1) - [b]\epsilon(k-2)$ to be injected at the summing node of the linear (i.e. quantization-free) filter, yielding an output noise spectrum

$$\frac{q^2}{12} |H_\epsilon(e^{j\Omega})|^2, \quad \text{where} \quad H_\epsilon(z) = \frac{1 - [a] \cdot z^{-1} - [b] \cdot z^{-2}}{1 - a \cdot z^{-1} - b \cdot z^{-2}}. \quad (5.12)$$

Error-feedback is most effective when a and b are close to integer values. This is the case e.g. for narrow-band low-pass filters (such as interpolation filters) which have poles clustered at $z=1$, so a and b will be close to 2 and -1, resp. The resulting error spectrum will be nearly white and its power highly insensitive to small changes in the parameters.¹⁴

For such filters two alternative structures have been proposed [369], which prove to be intimately related to error-feedback structures [318], [343]. The first of these splits up the parameters a and b into integer parts $[a]$, $[b]$ and decimal parts $a_1 := a - [a]$ and $b_1 := b - [b]$, introducing two quantizers in front of the multipliers a_1 and b_1 to allow for single-precision multipliers in combination with double-precision unit-delay elements. The resulting filter is basically a second-order error-feedback structure and as such performs equally well.

The second alternative [369] is based on the notion that a filter that has poles close to $z=1$ might profit from a change of origin in the complex z -plane, as much from the point of view of sensitivity as of roundoff noise. Introducing the complex variable $\hat{z} := z-1$ we can write

$$H(z) = \frac{1}{z^2 - a \cdot z - b} = \frac{1}{\hat{z}^2 + (2-a) \cdot \hat{z} + (1-a-b)} = \frac{1}{\hat{z}^2 - \hat{a} \cdot \hat{z} - \hat{b}} := \hat{H}(\hat{z}), \quad (5.13)$$

where $\hat{a} := a-2$ and $\hat{b} := b+a-1$.

This \hat{z} realization of the all-pole filter requires a realization of \hat{z}^{-1} , which is simply a delay with unity gain feedback. Such 'integrator' based structures (also known from digital incremental computers [367]) are studied in [367]-[378]. They can be interpreted as filter realizations using first-order error-feedback across each delay element [318]. Also, alternative arithmetics are considered (distributed, ROM/accumulator [338], [377]) in-

¹⁴Due to the smoothing of the resonance peak in the error spectrum, error-feedback will also significantly reduce the amplitude of possible limit cycles (though not the probability of their occurrence, since filters that are free of limit cycles may lose that property by applying error-feedback). Perfect error-feedback (i.e. without restricting the feedback coefficients to integers) would produce limit cycle-free filters but for the necessary double-precision quantizer(s) in the error loop. This is not surprising, since perfect error-feedback is just nothing more than double-precision arithmetic with a single-precision output [324].

cluding filter zeros (as well as poles) and even error-feedforward in order to optimally shape the error spectrum for such zeros [336].

To conclude this section, we mention some other papers on roundoff noise and its reduction. A special choice of structure, such as wave digital structures [379]-[389], may significantly reduce the quantization noise (as well as the parameter sensitivity). Various topics relating to roundoff noise are treated in [390]-[423], such as comparisons between different realizations, special arithmetics, sensitivity, scaling, simulation and measurement. This group contains contributions which do not necessarily fit into one of the main categories on quantization noise, i.e. statistics, optimization or reduction. Also some papers are included which study quantization and overflow stability as well as noise in special structures or even in a comparison between different solutions, but which cannot be designated as review articles such as [3]-[10].

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